NUMERICAL CALCULATION OF THE MOTION OF A GAS FROM A SURFACE EXPLOSION

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The motion of a gas by the normal impact of a high-speed body at the interface between a dense half-space and a vacuum is investigated numerically. The motion of the shock wave and the shape and distribution of the parameters of the gas dispersing in the vacuum are obtained. The motion is studied during the formation of a region with high pressure at the boundary with the vacuum of a gas occupying the half-space $z > 0$. The assumption of cylindrical symmetry relative to the z axis enables this three-dimensional nonsteady-state problem in the general case to be solved as a two-dimensional problem. For the corresponding one-dimensional problem, the numerical solution and, for certain gases also, the analytic solutions are well known and are considered in detail in [1]. As a result of solving the two-dimensional problem, profiles of the gasdynamic quantities are obtained which are similar to the solutions in the one-dimensional case and the result of the solution by a self-similar method. The cup-shaped surface of the shock wave front with a pressure gradient on it "focusses" the dispersing gas so that its velocity component normal to the surface $z = 0$ is greater by an order of magnitude than the component parallel to the surface of separation of the medium, and only at individual points is their ratio close to 0.4. Therefore, the dispersing gas is formed into the shape of a "jet", the pressure and density profiles on the axis of which have a shape similar to the one-dimensional problem of a brief shock, but in the plane $z = 0$ the pressure and density distributions are similar to the distributions of these quantities in the case of a powerful point explosion in an unbounded medium. The initial disturbance in the symmetrical problem being considered may be the result of either the normal impact of the body with a high velocity at the surface of the dense medium, or the consequence of the effect of a giant laser pulse, or some other process when a certain volume is formed with a high pressure at the interface between the dense medium and a vacuum, or with another low-density medium.

Let us consider the formulation of the problem. Because of the symmetry of the starting conditions relative to the z axis, the equations do not contain an angle of rotation around the z axis and the problem is solved in the half-space $\varphi = \text{const}$ with the cylindrical system of coordinates r, φ, and z.

To the left of the surface $z = 0$ there is a vacuum and to the right there is a homogeneous, cold, ideal gas with pressure $p = 0$, density $\rho = 1$, velocity along the z axis $u = 0$, velocity along the r axis $v = 0$ and adiabatic index $\gamma = 1.4$. All quantities are dimensionless, so that the gasdynamic equations do not contain additional constants. The starting region has the shape of a cylinder of height 2 and diameter 20, the bases of which are parallel to the surface $z = 0$, bisecting it and the axis of the cylinder lies on the z axis (Fig. 1). The values $p = 0.4$, $\rho = 1$, and $u = v = 0$ are assigned to the starting region. The boundary of the region at various instants of time is shown by curves 0 to 6 (Fig. 1).

We denote by the letters *a* and d the points of intersection of the boundary between the negative and positive directions of the z axis. On the intercept *ad* of the z axis, in view of the symmetry of the problem, a "wall" condition arises. We denote by the letters b and c the points of intersection of the r axis by the boundaries to left and right. For the starting region they coincide, then the point b is displaced above c and at the instant bc the wall condition arises. The calculation, taking into account perturbations penetrating

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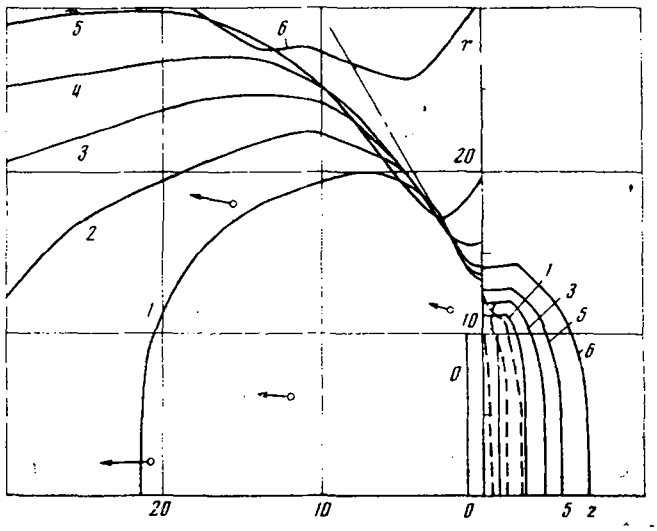

Fig. 1

through bc, showed the advisability of this simplification for the achieved accuracy of the solution, as the maximum change of the gas parameters in the perturbed region is less than 10%, and in itself is small. The section ab borders on the vacuum, and cd on the cold, dense, quiescent gas, i.e., $p=0$, $\rho=1$, and $u=v=0$.

The system of gasdynamic equations is solved in the region:

$$\frac{\partial \rho r}{\partial t}+\frac{\partial \rho u r}{\partial z}+\frac{\partial \rho v r}{\partial r}=0$$

$$\frac{\partial \rho u r}{\partial t}+\frac{\partial (p+\rho u^2) r}{\partial z}+\frac{\partial \rho u v r}{\partial r}=0 \qquad (1)$$

$$\frac{\partial \rho v r}{\partial t}+\frac{\partial \rho u v r}{\partial z}+\frac{\partial (p+\rho v^2) r}{\partial r}=p$$

$$\frac{\partial \rho (e+(u^2+v^2)/2) r}{\partial t}+\frac{\partial \rho u (e+p/\rho+(u^2+v^2)/2) r}{\partial z}+\frac{\partial \rho v (e+p/\rho+(u^2+v^2)/2) r}{\partial r}=0$$

where the internal energy e is related with p and ρ by the equation of state of an ideal gas $p=(\gamma-1)\rho e$. The scheme for Eq. (1) is formulated and solved numerically by the method described in [2, 3].

The condition at ab is achieved by the motion of the edges of cells returning to the vacuum side with velocity $\omega_0+2c_0/(\gamma-1)$, where ω_0 is the velocity normal to the boundary ab of the gas adjacent to the vacuum of the cell, and c_0 is the velocity of sound in it. The pressure, density and velocity at the boundary itself are assumed equal to zero.

In the conditions at cd the external pressure is assumed to be not exactly zero, but is assumed at every instant of the calculation to be equal to the pressure at the shock wave front, reduced by a factor of 10^4 which, with high accuracy, approximates an infinitely strong shock wave and facilitates the programming in comparison with an infinite ratio of the pressures. If desired, a ratio can be assigned which is greater by several orders, but even with the chosen values a compression in the shock wave of a factor of 5.96 is obtained, with a limiting value of 6.

Let us consider a difference scheme. In preparing the scheme for Eq. (1), it is taken into account that in contrast from [3], the problem being solved does not emerge from the steady-state cycle. Therefore, where necessary, averaging of the cell dimensions is carried out for adjacent instants of time, which proves to be an adequate measure, as the movement of a cell during one time step is small, and terms higher than the first order of magnitude of the cell displacement can be neglected. In the calculations, a limit is imposed on the step in time, in order that the displacement of the cell during this step is small in comparison with the size of the cell.

The chosen network differs from [3] in that the origin of the rays of the network is on the r axis and not on the z axis, which lengthens the numerical formulas. This change is made for convenience of calculation of the problem with radiation incident on the z axis. It was found that the problem of dispersion in vacuo is badly approximated in a network with the origins of the rays on the z axis. Unacceptably large parasitic velocities v originate during the first steps in time. In the network with origins of the rays on the

r axis, this effect does not arise, which shows the importance of the choice of the type of network for calculating the dispersion in vacuo. Obviously, the difference is associated with the fact that high velocities occur during dispersion and with the origin of the rays chosen on the r axis the motion of the cells along the rays takes place in the direction of the velocity, which leads to better conservation of energy and mass. If we substitute a gas with density of order unity for the vacuum to the left of ab then, as the calculations show, both networks are equally suitable for this problem.

The difference scheme is obtained by integration of Eq. (1) with respect to the cells, so that constant average quantities are obtained in them and the total energy, momentum and mass is conserved. The two-dimensional decay of a discontinuity at the boundary of the cells [3] is calculated by these quantities. The methods of averaging in the cells on the z axis and in those at a distance from it are different. Suppose that the velocity v is proportional to r close to the z axis, at which it vanishes in the case of the symmetrical problem. By averaging v over the cell adjacent to the axis with $\Delta z = 1$ and $\Delta r = R$, we obtain in the case of constant density ρ and the velocity v varying linearly from 0 to V, that the momentum and mass in the cell is

$$g = \int_0^R \frac{rV}{R} \rho r\, dr = \rho V \frac{R^2}{3}, \qquad m = \int_0^R \rho r\, dr = \frac{\rho R^2}{2},$$

whence the average velocity $v = g/m = {}^2/_3 V$.

In order to calculate the decay of the discontinuity, averaging should be carried out not over a cylindrical cell, as was done above, but over a parallelpiped. Then,

$$g = \int_0^R \frac{rV}{R} \rho\, dr = \rho V \frac{R}{2}, \quad m = \int_0^R \rho\, dr = \rho R, \quad v = \frac{g}{m} = \frac{V}{2}.$$

By averaging in the cylindrical case, a value is obtained for the velocity v at the boundary of the cell on the z axis, which is higher by a factor 4/3. When r – the diameter of the cell – is decreased, this effect disappears.

A similar comparison of averaging for energy gives a correction factor of 9/8 with v^2 in the expression for the total energy.

Control calculations showed that for the chosen network, the corrections do not significantly improve the solution.

In the case of strong rarefaction waves, impact is possible at the boundary of the cells in the "fan" of the rarefaction wave [4]. This would require refinement of the formulas in [2] for the decay of the discontinuity. Calculations were carried out to verify the effect of such refinement on the solution in the one-dimensional and two-dimensional problems of a brief shock. Refinement was found to be necessary only in the very rarefied region with a low pressure and density and remote from the shock wave. In this case, a slight improvement of accuracy was observed in the one-dimensional problem (the energy was better conserved), but in the two-dimensional case the changes were not noticeable, in view of the lower overall accuracy of the solution. Therefore, with the chosen law of motion of the network, refinement of the decay formulas was not carried out.

The best approximation is obtained in the case of motion when the maximum stable step in time is identical for all the cells obtained [3]. In practice, the size of the cell was chosen on the principle of equality of mass in the cells. The reference region was divided up into subregions with continuous solutions, in which the dimensions of the cells varied according to a law of the type of a geometrical progression, such that the dimensions of the cells of opposite boundaries were inversely proportional to the density in them. The separation into regions with boundaries at singularities of the solution makes it possible to obtain discontinuous solutions accurately, without approximating them to continuous functions.

In one-dimensional problems, the process of separation into regions of continuous solution can be automated [5]. In the two-dimensional case, this is difficult to achieve and it is necessary to assign regions of separation in the starting conditions.

With a pressure or velocity jump less than 0.1, the decay of the discontinuity is calculated by approximate formulas. In the contrary case, interraction was performed up to convergence with an accuracy of 0.5%.

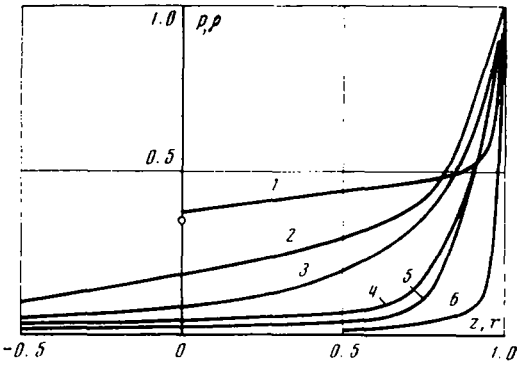

Fig. 2

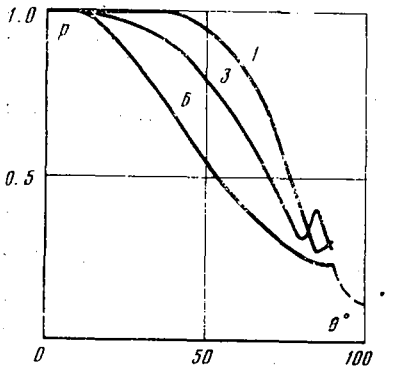

Fig. 3

Figure 1 shows the position at different instants of time of the perturbation boundary propagating from the starting region (rectangle 0). Curves 0-6 correspond to times t = 0.2, 3.5, 4.8. 6.3, 9.1, and 18.3. Curves 2 and 4 of the shock wave front are omitted.

The arrows show the velocity field for t = 9.1. The dashed lines mark the passage of the z component of the velocity u through zero for curves 1, 3, and 5. The boundary condition of the wall bc on the r axis affects the motion of the point b. A self-similar solution is obtained in [6] for the motion in the vicinity of b, and it is found in particular that the angle between the r axis and the boundary with the vacuum in it is equal to 27.7° for $\gamma = 1.5$. The chained line is drawn at an angle of 30° to the r axis. It can be seen that curves 1-5 touch it. The wall condition begins to appear in the last instants of time.

The part of the reference region for curves 2-6, which is not located in Fig. 1, contains gas of low density and pressure. The coordinate of the point *a* of the intersection of the boundary of the region with the negative direction of the z axis for curves 1-6, is equal to −21.4, −32, −43.6, −55.5, −71, and −156.

Below the point c and to the right of the r axis in Fig. 1 there is compressed gas with density ≈ 5, and to the left of the r axis there is a contact discontinuity with density ≈ 0.1. With a quite small size of the cells, it must be calculated exactly. It is possible, with cells which are not very small, to obtain a good approximation by taking the sizes of the cells corresponding to the equation for the optimum time step for the cells along both sides of the discontinuity [3].

Therefore, in order to increase the accuracy, the angle of inclination of the adjacent ray with the r axis was varied during the calculations, so that the optimum time steps for cells to the left and right of the r axis were equal. As a result, the solution changed only slightly. For example, curves 1-4 (Fig. 1) were calculated with reversal of the ray. Then the problem was solved with a distribution of rays fixed uniformly with respect to angle from the z axis, and a curve was obtained which almost coincided with 4, and curves 5 and 6.

The efficiency of the calculation with respect to expenditure of machine time in this case varied by more than a factor of 2. With reversal of the ray, the size of the cell at the intersection of the ray with the r axis is decreased, which affects the magnitude of the time step. A solution was obtained such that the density to the right of the r axis decreased sharply downwards from the point c and the contact discontinuity is small in size along the r axis. For example, for curve 6 a shift downwards from the point c by 7% of its distance to the z axis gives a density reduction by a factor of 10. The contact discontinuity occupies ~15% of the area of the hole for the emission of the gases at the surface z = 0. The density and pressure distribution along the r axis when t = 18.3 to the right of the contact discontinuity is shown by curves 1-6 in Fig. 2, where all quantities are normalized to their values at the shock wave front. The density coincides with an accuracy up to the error of the calculation with a profile in the wave of a point explosion in an unbounded medium.

The pressure differs from the case of a point explosion somewhat more, but when r = 0, the graph passes beside the point for a point explosion, which is located in Fig. 2 by the small circle on the axis of ordinate.

Curve 2 – the pressure, and curve 5 – the density, are on the z axis when t > 7.5. For comparison, Figs. 3 and 4 show the pressures and densities of the one-dimensional problem when $t \to \infty$. The velocity u on the z axis is linear with respect to z, and passes through zero when z = 0.5, i.e., it is similar to the one-dimensional case.

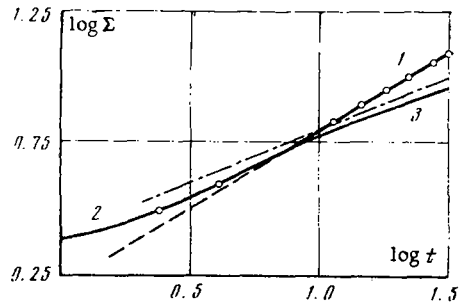

Fig. 4

In Fig. 3, the continuous line shows the change of pressure at the shock-wave front as a function of the angle θ between the ray from the origin of the coordinates at a point of the front and the z axis. Curves 1, 3, and 6 correspond to the times 2, 4.8, and 18.3; the dashed section is the pressure at the continuation of the line of the shock front on the rarefaction wave adjoining the side $z < 0$. The nature of the pressure buildup at 80° is explained by the presence of carbon in the starting region.

If the radius of the original cylindrical region tends to ∞, then the problem converts to the one-dimensional problem. Its solution is known, but for comparison of the result by self-similarity it was calculated with a one-dimensional variation of this same scheme, and the corresponding curves are plotted in Fig. 4. At first, the motion of the shock front along the z axis is identical for both problems and passes along curve 2. Then the one-dimensional motion emerges on the straight line 1 with slope 0.6 and the two-dimensional motion emerges on the straight line 3. The chained line is plotted with a slope of 0.4, which corresponds to a point explosion in an unbounded medium. Unfortunately, the accuracy of the calculation is inadequate for determining the dependence of the self-similarity index on γ.

This same problem has been solved with the starting region in the form of a hemisphere. The results differ little from those demonstrated above. For investigating the self-similar solutions, this initial shape was found to be less suitable, as the self-similar distribution of the quantities for it is ascertained to be several times longer than for a thin disk.

The errors, judging by controlled totals for mass and energy, increased with time, were independent of the size reduction of the time step and amounted to 11 and 15% respectively when $t = 18.3$.

The network consisted of 34 rays and 21 points on each of them. This, together with the program, required 13 memory sheets. The time of calculation of 100 time slices was 4 min. The program is compiled in analog, retransmitted through a "link" system with an α translator. This gave a computer time economy of a factor of 3.2 in comparison with the usual ALGOL translator. The capabilities of the computer permit the number of cells to be increased by a factor of 8, but the calculation time for the problem increases as the square of the number of points.

Curve 6 (Fig. 1), with a stability reserve of 0.6 in the time step and a uniform distribution of rays with respect to angles, was obtained at the 2400th time step and ~2 h of computer time were expended. The use of a mobile grid was found to be effective, as was a variable step in space. This same number of time steps for the one-dimensional problem with a grid of 40 cells enabled the motion of the shock-wave front to be calculated to distances of $2 \cdot 10^4$ diameters of the starting region with an accuracy of 0.5% and required 5 min of calculation.

The necessity for a reserve of stability is confirmed by the following example. In the calculation of the stationary shock wave, regions 1 and 2 of the calculation are distinguished ahead of the wave and behind it. The network in them is uniform. The boundary between the regions trails behind the shock-wave front, and in region 1 the quantities at the boundary with 2 are taken from the nearest cells to region 1, as perturbations from 2 do not penetrate it. During the calculation, with the maximum stable step Δt in the initially unperturbed region 1, parasitic waves originate with increasing amplitude. These waves are not observed in the calculation with a stability reserve.

A stability criterion was obtained in [2] for linearized equations. In the nonlinear case, the criterion may be violated.

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